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PROJECT APOLLO

RTCC REQUIREMENTS FOR MISSIONS F AND G: SELENOGRAPHIC-MNBY VELOCITY TRANSFORMATION FORMULATION

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SUMMARY

This document presents the formulation and implementation procedures for use in the RTCC for transforming velocity vectors from the selenographic coordinate system to the mean nearest Besselian year (MNBY) system. The formulation includes equations for computing the lunar angular velocity vector.

INTRODUCTION

The selenocentric MNBY system is moon centered and has its axes parallel to the following: the X-axis is in the mean earth equatorial plane and through the mean vernal equinox at the beginning of the nearest Besselian year; the Z-axis is north through the mean earth spin axis; and the Y-axis is on the mean earth equatorial plane and forms a right-handed orthogonal system.

The moon fixed selenographic system is defined as follows. The X-axis is fixed in the moon's equatorial plane and points approximately toward the center of the earth; the Z-axis is fixed in the moon through the moon spin axis; and the Y-axis is in the equatorial plane and completes the right-handed orthogonal system.

The interpolated precession-libration matrix (L) used to transform the velocity is also used to transform position vectors. If R_M is a position vector in the selenocentric MNBY system and R_G is a position vector in the selenographic system, then

$$R_M = L^T R_G$$

and

$$R_G = LR_M \text{ .}$$

VELOCITY VECTOR TRANSFORMATION

The inertial MNBY velocity, $\dot{R}_M$, of a vehicle with a fixed selenographic position, R_G, is computed using the following equation

$$\dot{R}_M = L^T(W \times R_G)$$

where W is the moon's angular velocity expressed in the selenographic system.

The following general equations can be used to compute the inertial velocity for a vehicle moving relative to the selenographic system at velocity $\dot{R}_G$ or for the reverse computation.

$$\dot{R}_M = L^T(\dot{R}_G + W \times R_G)$$

$$\dot{R}_G = L\dot{R}_M - W \times R_G$$

COMPUTATION OF W

The following equations are used to compute the angular velocity vector, W. These equations account for the rate of change of Ω, the longitude of the mean ascending node of the lunar orbit measured in the ecliptic plane from the mean equinox of date, and M, the geocentric mean longitude of the moon measured in the ecliptic plane from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit.

The computations neglect the small effect of the rates of precession, obliquity of the ecliptic, and physical libration. The inclination (i) of the lunar orbit to the ecliptic is constant and is $1°32'39''$ (ref. 1).

Prelaunch initialization:

1. Input year of launch, YOL, and day of launch, DOL.

2. Compute Julian centuries from January 0.5, 1900 to midnight prior to launch:

$$TIL = \text{integral part } [(YOL - 1901)/4]$$

$$TB = (YOL - 1900)365 + TIL + DOL$$

$$TM = TB/36525$$

3. Compute time rates of angles, $\dot{\Omega}$ and $\dot{M}$ in rad/hr:

$$\dot{\Omega} = [-1934.1420083 + 0.004155556TM + (0.6666667 \times 10^{-5})TM^2]RATE$$

$$\dot{M} = [481267.8831417 - 0.002266667TM + (0.5666667 \times 10^{-5})TM^2]RATE$$

where $RATE = \pi/(180 \cdot 36525 \cdot 24) = 1.9110212776572292 \times 10^{-8}$ and converts $\dot{\Omega}$ and $\dot{M}$ from deg/Julian century to rad/hr.

4. Compute constant terms:

$$K1 = \dot{\Omega} \cos i$$

$$K2 = \dot{\Omega} \sin i$$

5. Compute W_Z in rad/hr:

$$W_Z = \dot{M} - \dot{\Omega} + K1$$

Computation of W_X and W_Y at any time t during mission:

1. Input hours from midnight prior launch, H.
2. Compute Julian centuries from January 0.5, 1900 to anytime t:

$$T = (TB + H/24)/36525$$

3. Compute $M - \Omega$ in deg:

$$M - \Omega = 11.2508889 + 483202.02515T - 0.003211111T^2 - (0.0333333 \times 10^{-5})T^3$$

4. Compute W_X and W_Y in rad/hr:

$$W_X = K2 \sin (M - \Omega)$$

$$W_Y = K2 \cos (M - \Omega)$$

REFERENCE

1. Roth, H.: Description of Two Modified Versions of the Selenographic Coordinate Transformation Program (Phase I). Report No. 3400-6059-RU000, July 21, 1965.